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Modelling the Spatial Spread of COVID-19 in a German District using a Diffusion Model

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Introduction

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- reaction-diffusion model of spatial COVID spread, especially in early stages (population almost entirely susceptible at the start)
- use data down to local level for district of Birkenfeld during winter 2020/21

Spatial discretization

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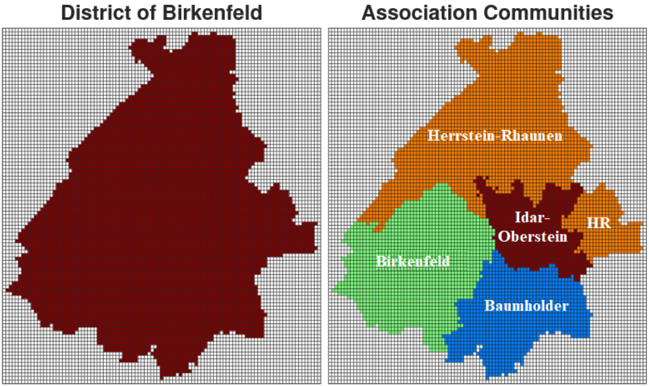

SEIR model

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$$
\begin{aligned}
S' &= -\frac{\beta(t)}{N} SI & S(t_0) &= S_0 > 0,\\
E' &= \frac{\beta(t)}{N} SI - \vartheta E & E(t_0) &= I_0 > 0,\\
I' &= \vartheta E - \gamma I & I(t_0) &= I_0 > 0.\\
R' &= \gamma I & R(t_0) &= R_0 \geq 0.
\end{aligned}
$$

with transmission rate β, latency rate ϑ, recovery rate γ

Model setup

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- consider a corresponding spatial area Ω in a time span $\mathcal{T}$
- searching for $u : V \to \mathbb{R}^m$ with $V = \Omega \times \mathcal{T}$, which is twice cont. diff'able on Ω and once cont. diff'able on $\mathcal{T}$, and solves

$$
\begin{aligned}
\partial_t u &= \kappa \Delta_{x,y} u + f(u)\,, && \\
u &= u_0\,, && t = 0\,, \\
\partial_\nu u &= 0\,, && (x,y) \in \partial\Omega\,.
\end{aligned}
$$

Model setup

- plugging in the *SEIR*-model and by $R = N - S - E - I$, we omit one equation
- transmission time-dependent $\beta(t)$, here: piecewise constant function with β_i, dependent on current social restrictions
- relative share $s = S/N$, $e = E/N$, $z = I/N$

Model setup

- we yield

$$
\begin{aligned}
\partial_t s &= \kappa\,\Delta_{x,y} s - \beta(t) s z, & s(t=0) &= s_0(x,y),\\
\partial_t e &= \kappa\,\Delta_{x,y} e + \beta(t) s z - \vartheta e, & e(t=0) &= e_0(x,y),\\
\partial_t z &= \kappa\,\Delta_{x,y} z + \vartheta e - \gamma z, & z(t=0) &= z_0(x,y)
\end{aligned}
$$

  for $(x,y) \in \Omega$, and $\partial_\nu u_j = 0$ for $(x,y) \in \partial\Omega$.

- for the total population N, we also have to solve a PDE:

$$
\partial_t N = \kappa \Delta_{x,y} N
$$

Parameter estimation

- parameter estimation of $\theta = (\beta_i, \kappa, \delta, u_{2,j})$ by adjoint method and Metropolis algorithm
- solve the PDEs using either Crank-Nicholson scheme, FEM (finite element method), ...
- results comparing Metropolis algorithm to adjoint method and plain *SIR*-model

Results

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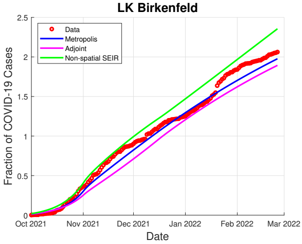

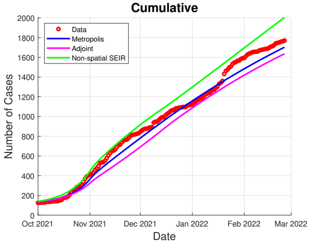

Abbildung 2: Optimization with the various methods for the district of Birkenfeld with $w_0 = 1$ and $w_1 = 10^{-5}$.

Results

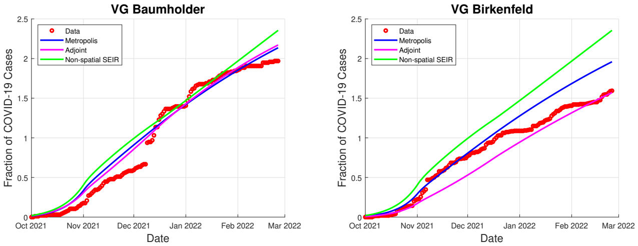

Abbildung 3: Optimization with the various methods for the lower level administrative units with $w_0 = 1$ and $w_1 = 10^{-5}$.

Results

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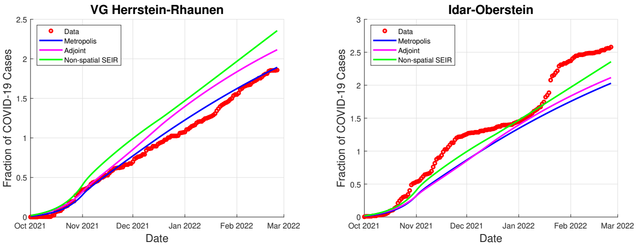

Abbildung 4: Optimization with the various methods for the lower level administrative units with $w_0 = 1$ and $w_1 = 10^{-5}$.

Conclusion

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- identification of several parameters
- decent fit of the model compared to medical data
- working well especially in initial phase

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Thanks for your attention!

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Informatik

Förderung

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RheinlandPfalz

MINISTERIUM FÜR WISSENSCHAFT UND GESUNDHEIT